ETHNOMODELLING: FRACTAL GEOMETRY ON THE DOOR ORNAMENT OF THE SUMENEP PALACE USING THE LINDENMAYER SYSTEM

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Abstract

The Sumenep Palace area is one of the cultural heritage relics of the Duke of Sumenep, which is still preserved today. This research aims to reveal the fractal model of door ornament in the Koneng Office of Sumenep Palace. A qualitative method with an ethnography type is used. Data were collected through initial observation, literature study, interviews, and documentation. Analysis of the fractal form is done with the Lindenmayer system method and constructed with the help of the L-Studio application. The results of the study revealed the existence of fractal structures on the door ornament of the Sumenep palace office. The length, angle, and ratio of each part of the ornament influence the fractal shape. The findings of this study can be utilized in learning mathematics in lectures, such as fractal geometry and computational geometry.

Keywords: Ethnomathematics, Ethnomodeling, Sumenep Palace, Lindenmayer System

1. Pendahuluan

Ethnomathematics is one of the areas of mathematics research that many researchers are now starting to explore. That is evidenced by the significant increase in exploratory studies in ethnomathematics in the last ten years (Pradana et al., 2022; Rusli & Safaah, 2023). The impact of this increase has led to an increase in derivative research related to ethnomathematics, such as the development of ethnomathematics-based learning tools (Sumiyati et al., 2018) to cognitive research wrapped in ethnomathematics nuances (Noto et al., 2018). As such, ethnomathematics research has brought new colour to the field of mathematics education research.
Ethnomathematics comes from three Greek words, namely *ethno*, *mathema*, and *tics* (Orey & Rosa, 2021; Rosa & Orey, 2022d). The term ethnomathematics was initiated by D’Ambrosio in 1948 in opposition to Western mathematics, which was considered too dominating, so mathematics developed by cultural groups was considered unmathematical (Umbara et al., 2021). In addition, ethnomathematics can be defined as a program that links mathematical concepts with cultural groups (Alghar & Marhayati, 2023). Ethnomathematics is also defined as the bridge that connects mathematics to the cultural sphere (Alghar et al., 2022, 2023). So, the definition of ethnomathematics is always related to the mathematical concepts used, applied, and inherited within a cultural group.

Ethnomathematics research is increasingly developing and specific with the existence of a new domain within it, namely ethnomodelling (Rosa & Orey, 2013). Ethnomodelling or ethnomodel is a study that examines the mathematical ideas and procedures that indigenous people apply, develop, and use in their daily lives (D’Ambrosio, 2015; Oliveira et al., 2021). Ethnomodelling is also defined as the study of mathematical modelling used by cultural groups that have been passed down through generations and applied in the life of the group (Rosa & Orey, 2022a, 2022c). That means that ethnomodelling research can be interpreted as a subset of the ethnomathematics research field, as Figure 1 shows.

Based on Figure 1, the scope of ethnomathematics research is an intersection of the fields of mathematics, mathematical modelling, and cultural anthropology (Rosa et al., 2016). At the same time, the scope of ethnomodelling research is an intersection of the fields of mathematical modelling, ethnomathematics, and culture (Rosa & Orey, 2022a, 2022c). That means that ethnomodelling is a more specific study of ethnomathematics that discusses mathematical modelling in a cultural setting. Furthermore, the discussion in ethnomodelling is not only a dialogue between mathematical modelling and
culture but also examines the values and meanings of a culture (Rosa & Orey, 2022b; Umbara et al., 2021). Thus, the output of ethnomodelling provides an understanding for the reader to appreciate and value the cultural values developed and applied in mathematical modelling by the cultural group.

Various studies related to ethnomodelling have been conducted by researchers from multiple countries, with diverse mathematical model findings. Umbara et al. (2023) looked at mathematical modelling in the Cigugur community in determining good dates for building a house. Santos & Madruga (2021) explore the mathematical modelling that Brazilians do when harvesting chocolate. Orey & Rosa (2015) formulated the catenary model on the artefact wall of the Colégio Arquidiocesano in Ouro Preto. Orey & Rosa (2020) looked at mathematical modelling in different cultures in Nepalese society.

However, ethnomodelling research is still dominated by cultures from abroad. As described by Rusli & Safaah (2023), ethnomodelling studies from 2012-2022 were dominated by research from Brazil, Argentina, and African countries. That means that research on mathematical modelling from Indonesian culture is still minimal. In addition, much of the mathematical modelling explored is still done in the fields of algebra, number theory, and calculus. Thus, mathematical modelling in terms of geometry is still minimal. This research tries to fill the gap by exploring ethnomodelling from the field of geometry.

One of the cultural heritage objects that has historical and cultural value and is still preserved is the Sumenep Palace. The Sumenep Palace is located in the centre of the Sumenep Regency on Madura Island, East Java (Herawati, 2014). Keraton Sumenep is a building that became the centre of the government of the Duke of Sumenep, which was completed in 1762 AD during the leadership of Ratu Tirtonegoro (Halim & Royandi, 2022; Herawati, 2014). The construction of the Sumenep palace was assisted by an architect of Chinese descent named Lauw Piango. With the help of Lauw Piango, Ratu Tirtonegoro succeeded in building the government centre of the Duke of Sumenep with a Chinese, European, and Sumenep-style building style (Halim & Royandi, 2022; Siddiq & Billa, 2023).
One of the buildings that stands in the Sumenep Palace area is the Koneng Office. In its time, this office was used as the workspace of the King or Queen of Sumenep. (Halim & Royandi, 2022). However, after the royal reign ended, this building was utilized to store royal collection objects, such as large mirrors and andong. (Herawati, 2014). Although it has not been used for a long time, the shape of the building and the ornaments that adorn the Koneng office are still beautiful and well-maintained until now.

One of the ornaments in the Koneng office that has cultural acculturation value is the ornament on the office door. As the result of the initial observation, this ornament adorns the door of the Koneng office in 4 parts. Each part of the ornament on the door looks harmonious and repetitive. The size of the trim appears balanced and cohesive. In addition, the yellow and red colours on the trim signify Chinese nuances (Halim & Royandi, 2022). Therefore, this ornament has characteristics that lead to fractal geometry, which is a repetitive shape but still in order.
Fractal geometry is one of the fields in non-Euclid geometry. If Euclid geometry talks about mathematics in one, two, or three-dimensional space, then fractal geometry talks about mathematics in fractional dimensional space, such as 1/2 or 3/2 (Widodo, 2021). That means that the dimensions used in fractal space will be different from Euclid geometry, so it is categorized in non-Euclid geometry. Therefore, the method used in constructing fractals cannot be done with the Euclid geometry method (Alghar & Marhayati, 2023; Widodo, 2021). There needs to be another method that is more specific to building fractal shapes in non-Euclid geometry. One method that can be used to make fractals is the Lindenmayer system method.

The Lindenmayer system or l-system is a rewriting method developed by Astrid Lindenmayer (Prusinkiewicz & Hanan, 2013). It uses computationally encoded basis functions and geometry transformations. The L-system constructs a recursive rewrite by using strings of functions that have been defined in the original position. (Alghar & Marhayati, 2023; Juhari & Alghar, 2021).

In general, an l-system is defined as a tuple $G = (V, \omega, P)$, with $V$ as a finite set, $\omega$ as an initial function on $V$, and $P$ as a set containing production rules (Bernard & McQuillan, 2021). The deterministic l-system has the rule $K \to x$, for every $K \in V$ with $K$ being the initiator. Whereas $x$ in the string $V$ is interpreted as the generator of $K$. With $K$ as a member of $V$ and $K$ mapped exactly one to $x$, it follows that every $K$ in $V$ is also mapped to $x$ (Bernard & McQuillan, 2021; Prusinkiewicz & Lindenmayer, 2012).

Some research using the Lindenmayer system method is still done in pure mathematics research. Alghar (2020) and Juhari & Alghar (2021) used the l-system to model the plant stem three-dimensionally. Fadhilah (2020) used the l-system in two-dimensional rod modelling. Yasar et al. (2009) applied the l-system to the design of scaffolding cell systems for accessing nutrients. Prusinkiewicz & Lindenmayer (2012) used the l-system to construct Hilbert and snowflake curves. However, these studies only examine the use of the l-system in plant objects and mathematical curves. In other words, there is not much research that uses the l-system to build fractal shapes in the cultural sphere.

Based on the description that has been presented, ethnomodelling studies are still dominated by various studies originating from abroad. In fact, as one of the countries with diverse cultures, Indonesia has a myriad of cultural potential that can be studied in the field of ethnomodelling. On the other hand,
the ornament on the door of the Koneng office in the Sumenep palace area is one of the cultural artefacts that is evidence of the acculturation of Chinese, Dutch, and Sumenep cultures. Its irregular shape resembles itself, and recursive makes the ornament have characteristics that resemble fractal shapes. In addition, research that uses the l-system to see fractal conditions in cultural buildings is still minimal. Therefore, this ethnomodelling research aims to model the fractal geometry shape of the Sumenep palace office door ornament using the Lindenmayer system.

2. Method
This research uses a qualitative method with an ethnographic approach. The flow in this research consists of six stages, which are explained as follows. 1) Introduction: in this stage, the research theme, research object, and research location are determined. The theme of this research is ethnomodelling, with the object of research being the ornament on the door of the Koneng office in the Sumenep palace area. The location of the research was conducted in the palace of Sumenep, East Java province, Indonesia. 2) Problem formulation: the researcher determines the problem to be researched, which is modelling the fractal form on the ornament of the Koneng office door with the Lindenmayer system method.

Data collection: this part of data collection is done in four ways, namely observation, literature study, interviews, and documentation. Initial observations were made by visiting the Sumenep palace area, followed by measuring the length and angle of the ornament. Literature studies are conducted by reviewing various books, section books, and articles related to ethnomodelling, the history of the Sumenep palace, and Chinese symbols. Interviews were conducted with Mr. Riananta as a special guide from Disbudporapar UPT Muesum Keraton Sumenep. Documentation is done by recording the results of measurements and photographing objects that become research studies.

Data Analysis: data from observations, interviews, documentation, and literature studies were analyzed using method triangulation. The measurement results of the length and angle of the ornaments were reduced, followed by computational analysis with the help of the L-Studio application. The result of L-Studio is a visualization of the fractal shape that has been built with the Lindenmayer system method. 5) Results and Discussion: L-system construction results, visualization results, and literature study results are explained and discussed in this section. 6) Conclusion: this section concludes
related research findings. The stages in this research are described in Figure 3.

3. Results and Discussion

Ornament Measurement Results

This section explains the measurement results of the length and angle of the ornament. The length measurement was done with the help of a ruler. At the same time, the angle measurement is done with a protractor. The ornament measurement process is shown in Figure 5.

The measurement results were then recorded and sketched on a millimetre block book. Then, the sketch is given a colour and name at each angle to make it easier to analyze the measurement results. The measurement results are shown in Figure 6.
Based on Figure 6, each corner is marked with a name and number. That is to make it easier for researchers to analyze the measurement results. The length between points has a multiple of 1.5 cm. So, researchers use a ratio of 1.5 cm to measure the size of each part of the ornament. In addition, the use of a 1.5 cm ratio is done to facilitate analysis in visualization using L-Studio. The results of the length measurement in each part are shown in Table 1. While the results of the angle measurement, it was found that the angle in each part of the ornament is 90°.

![Table 1](image)

**Table 1. The results of the length measurement in each part of the ornament**

<table>
<thead>
<tr>
<th>The ornamental part represented in Figure 6</th>
<th>Length</th>
<th>Ratio (1,5 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[A_1A_2], [A_2A_3], [B_1B_2], [C_1C_2], [C_2C_3], [D_1D_2], [D_2D_3], [E_1E_2]</td>
<td>1,5 cm</td>
<td>1</td>
</tr>
<tr>
<td>[F_1F_2], [F_2F_3], [G_1G_2], [H_1H_2], [H_2H_3], [I_1I_2], [I_2I_3], [J_1J_2]</td>
<td>3 cm</td>
<td>2</td>
</tr>
<tr>
<td>[K_1K_2], [K_2K_3], [L_1L_2], [M_1M_2], [M_2M_3], [N_1N_2], [N_2N_3]</td>
<td>4,5 cm</td>
<td>3</td>
</tr>
<tr>
<td>[A_8A_7], [B_1B_2], [C_1C_2], [D_1D_2], [E_1E_2], [F_1F_2]</td>
<td>9 cm</td>
<td>6</td>
</tr>
<tr>
<td>[G_1G_2], [H_1H_2], [I_1I_2]</td>
<td>19,5 cm</td>
<td>13</td>
</tr>
<tr>
<td>[K_1K_2]</td>
<td>22,5 cm</td>
<td>15</td>
</tr>
</tbody>
</table>

**Coding Results with Lindenmayer System**

Coding with the L-system method on ornaments is based on the length of the trim and the size of the angle. Researchers created parameters using the ratios in Table 1 that will be used in the coding. In addition, coding with the L-system uses several symbols, as described in Table 2.

![Table 2](image)

**Table 2. Symbols used in the L-system**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>The Meaning of Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F)</td>
<td>Step one time forward by 1.5 cm.</td>
</tr>
<tr>
<td>(+\theta)</td>
<td>Rotate clockwise by (\theta) degrees.</td>
</tr>
<tr>
<td>(-\theta)</td>
<td>Rotate counter-clockwise by (\theta) degrees.</td>
</tr>
<tr>
<td>(\rightarrow)</td>
<td>Mapping</td>
</tr>
<tr>
<td>(m)</td>
<td>Rotate by 45 degrees.</td>
</tr>
<tr>
<td>(*)</td>
<td>The multiplication operation</td>
</tr>
<tr>
<td>(K)</td>
<td>Set (K) with specific production rules.</td>
</tr>
<tr>
<td>(L)</td>
<td>Set (L) with a specific production rule.</td>
</tr>
</tbody>
</table>

Next, the researcher created production rules on the l-system based on the
ratios in Table 1, the sketches in Figure 6, and the symbols in Table 2. The results of the l-system coding are shown in the following explanation.

**Parameter:** Define $m=45; \alpha=7$

**Initiator (w):** $[-(m)K_{n+1}][+(m)K_{n+1}][-(3*m)K_{n+1}][+(3*m)K_{n+1}]$

**Iteration (n):** 4

**Production Rules:**

$K_{(n+1)} \rightarrow [F-(2*m)F+(2*m)F+(2*m)F(m)+(2*m)F+(2*m)F][L_{n}][+L_{n}][-L_{n}]]$

$L_{(n+1)} \rightarrow [F+(2*m)F-(2*m)F-(2*m)F(m)-(2*m)F-(2*m)F[K_{n}]]$

The constructed production rule consists of two rules. The initial production rule is formed by $K_{(n+1)}$, which will eventually map to $L_{n}$. At the same time, the next production rule is started by $L_{(n+1)}$, which will ultimately map to $K_{n}$. With $n$ being the number of iterations, there will be an iterative function between $K_{(n+1)}$ and $L_{(n+1)}$ 6 times. The screen display of the l-system production rules in the L-studio application is shown in Figure 7.

![Figure 7. Display of production rules in the L-Studio application](image)

**Visualization Results with Lindenmayer System**

The basic shape of the ornament is formed by the first ($K_{(n+1)}$) and second ($L_{(n+1)}$) production rules. The first production rule is shown in Figure 8a, and the second production rule is shown in Figure 8b. In general, the shapes of the images shown in both production rules are the same, but the production rules have different descriptions. That is because the directions used in the two production rules are opposite to each other.

![Figure 8. Visualisation results: (a) production rule $K_{(n+1)}$, (b) production rule $L_{(n+1)}$](image)
After the two production rules run, the string will continue in the first iteration, second iteration, and third iteration until the fourth iteration. The visualization results of the four iterations are shown in Figure 8.

![Visualization results](image)

Figure 9. Visualization results: (a) first iteration, (b) second iteration, (c) third iteration, (d) fourth iteration

Based on Figure 9a, it can be seen that the constructed image, like Figure 8, is rotated in four opposite directions. It happens as a result of the production rules $K_{(n+1)}$ and $L_{(n+1)}$ being subjected to the mapping on the initiator ($w$), which is $[-(m)K_{n+1}]+(m)K_{n+1}][-3(m)K_{n+1}]+(3*m)K_{n+1}]$ with one iteration. In other words, the production rule will rotate $45^\circ$ and $135^\circ$ clockwise and $45^\circ$ and $135^\circ$ anti-clockwise.

Meanwhile, Figures 9b, 9c, and 9d show that the production rules and initiators in the run l-system have formed fractals. That is because $K_{(n+1)}$ and $L_{(n+1)}$ are repeated 2, 3, and 4 times. In addition, the branching direction is also set in the initiator ($w$), namely $[-(m)K_{n+1}]+(m)K_{n+1}][-3(m)K_{n+1}]+(3*m)K_{n+1}]$. That makes the branching always lead to $45^\circ$ and $135^\circ$ clockwise and $45^\circ$ and $135^\circ$ anti-clockwise. Thus, the more iterations ($n$) performed, the larger the branching will be.
To validate the visualization results, the researchers compared the visualization results with the object of research, namely the ornaments at the Koneng Office. That is useful to see whether the l-system construction built has produced the appropriate shape or not. In addition, comparing the visualization results aims to see whether or not there are errors in the visualization results. If there are errors, then the researcher will reconstruct the coding of the l-system that has been made before. A comparison of the visualization results with the ornament that became the object of research is shown in Figure 10.

![Comparison of visualization results](image)

Figure 10. Comparison of visualization results: (a) ornaments in Koneng Office, (b) Fourth visualization result

**Cultural Values in the Door Ornament of the Sumenep Palace Office**

**Geometric Values of Koneng Office Door Ornament**

The results of the visualization shown in Figure 9, as well as the production rules constructed in Figure 7, show that the Chinese ornaments in the Koneng office can be built using the Lindenmayer system method. Furthermore, Figure 9 shows that the ornament has a shape that repeats itself, is recursive, and has regularity. That illustrates that the properties of fractal geometry have been fulfilled from the ornament. That means that, from a mathematical point of view, the ornament can be classified as an ornament with non-Euclid geometry.

In terms of the literature review, some researchers point out that one type of Chinese-style ornament can be categorized into geometric ornaments (Dye, 2012; Xu et al., 2020). More specifically, some geometric-style Chinese decorations have repetitive, regular, opposing, and indefinite types (Beer,
The opposite form means that there is a need for balance in life, such as the existence of good and bad that balance each other's lives (Beer, 2004). The repetitive and unlimited form means that happiness is always sustainable and endless. While the regular shape means that the good done needs to be organized and on a consistent path (Dye, 2012; Welch, 2013; Wen, 2011). This statement is in line with what Mr. Riyananta explained in the interview, that the ornament is a form of depiction of the harmonious life of the Madurese and Chinese communities during Panembahan Sumolo's era.

'T' Shape Motif on Koneng Office Door Ornament

Figure 11. T-shape motif: (a) In the production rules, (b) In the second iteration, (c) In the door ornament of Koneng Office

The visualization results on the first to fourth iteration rules shown in Figure 11 have a shape like the letter T. Some literature states that the T shape is an archetype that has been used by the Greeks and developed until now (Beer, 2004; Dye, 2012). T-shape patterns are usually applied to the edges as decoration (Alghar & Marhayati, 2023).

Figure 12. T-Motif Shapes in Meander Ornaments

Beer (2004) and Wen (2011) explained that Chinese engravers tend to dislike the emptiness on the edge of an object. Many Chinese engravers decorated it with T-shapes, L-shapes, and S-shapes to fill the void (Lee-Kalisch, 2018). These shapes can be decorated until they are linked to each other. Furthermore, the interconnected T-shape means the hope for a long life for the decorated object and its owner (Beer, 2004; Dye, 2012; Wen, 2011). In other words, the shape of the T is not only meaningful as a decoration but there are hopes to be conveyed to the owner.
Swastika Symbol on Koneng Office Door Ornament

The visualization results of the first and second production rules shown in Figure 9 have a swastika symbol. Some literature states that the swastika symbol is one of the oldest symbols used in human civilization (Beer, 2004; Mohamed & Mostafa, 2022). The word swastika comes from the Sanskrit word sv-asti, which means goodness and happiness (Alghar & Marhayati, 2023; Beer, 2004; Dye, 2012). The swastika symbol is shown in Figure 13.

![Swastika Motif](image1)

Figure 13. Swastika motif: (a) In the first iteration, (b) In the second iteration, (c) On the door ornament of the Koneng Office

Furthermore, the direction shown on the swastika has different meanings. If the direction of rotation is clockwise, then the symbol means the masculine nature of God. If the direction of rotation is anti-clockwise, then the symbol depicts the feminine nature of the Goddess. (Beer, 2004; Mohamed & Mostafa, 2022; Zidan, 2020). The swastika symbol also means the sacred heart of the Buddha (Sattarnezhad et al., 2020). Some literature also interprets the Swastika symbol as ten thousand virtues, longevity, and unlimited blessings (Dye, 2012; Wen, 2011).

![Swastika Shapes](image2)

Figure 14. Swastika shapes

The presence of the swastika symbol as an ornament on the door of the Koneng Office seems to be in line with the swastika symbol on the gate of the Jamik Mosque in Sumenep (Alghar & Marhayati, 2023; Fajariyah, 2021). This harmony is shown by the installation of swastika symbols as door and gate ornaments. If the swastika symbol on the Sumenep Jamik Mosque Gate depicts messages of goodness from Chinese culture, then the same seems to be conveyed by the swastika symbol on the Koneng Office door.
4. Conclusions

Based on the results and discussion, Chinese-style ornaments on the door of the Koneng Office in the Sumenep palace area fulfil the concept of fractal geometry. The length and angle of the trim influence the shape of the built fractal. Fractal formation is assisted by the Lindenmayer system method with a slope of 45°, a ratio of 1.5 cm, and four iterations.

This research is still limited to ethnomodelling studies with the specification of exploration of fractal shapes on ornaments at the Pintu Kantor Koneng Sumenep. The researcher hopes that further research can bring the results of this study to the realm of application in the field of mathematics learning in lectures, especially in computational geometry and mathematical modelling materials. In addition, researchers hope to add thickness variables in future studies so that the resulting visualization can be more in line with the original object.

References


Tuhan Yang Maha Esa dan Tradisi.


